FAILURE CRITERIA
FOR VISCOELASTIC MATERIALS

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This report reviews work performed since 1 March 1967 under the research grant NsG-172-60, GALCIT 120. Because the end of the semi-annual reporting period -- January-July -- bisects the intensive summer efforts, we have deferred this report in order to include results obtained during the last two months. Some of the earlier work has been submitted for oral or journal presentation as listed in the bibliography[1-4].

The work to be discussed in the following pages covers:

I Material Characterization
- a) Wave propagation in a rod
- b) Thermorheological simplicity in uniaxial tension failure

II Stress Analysis of Crack Geometries
- a) Experimental evaluation
- b) Large deformation numerical analysis

III Crack Propagation in Sheet Geometries
- a) Theoretical work
- b) Experimental work

Results obtained in these subjects will now be discussed in the order listed.

I MATERIAL CHARACTERIZATION

As an outgrowth of examining the mathematical representation of linearly viscoelastic materials and their use in stress analysis - see the last semi-annual report[5] - the problem of wave propagation in viscoelastic materials has been considered and brought to conclusion. It has been shown how the initial temperature affects wave propagation and that a step pulse travelling initially with the glassy wave speed changes shape through a transition with the result that the final wave shape is again a step pulse travelling with the lower wave speed corresponding to the rubbery modulus.

For the purpose of fracture mechanics it can be shown that only a limited amount of the material response spectrum will affect the motion of a crack running at high velocity.

Using measured material properties of Hysol 8705, the response of a bar to a step displacement at the end was calculated. The result is summarized in Figure 1 which shows the displacement history at various stations down the rod. A step travelling with glassy velocity is represented by a step at $\xi = 0$ and a step travelling with the rubbery speed is represented by a step at $\xi = 1$.

In connection with the compilation of characterization data on Solithane 113, some supplementary tests were performed on failure data under constant rates of elongation. This was done to check on some anomalous appearing points in the already existing data. When finally analyzed, it turned out that the tensile data could not be superposed according to the shift procedure commonly reasonably applicable to failure data like for modulus data. The implication of this may be twofold:

Applicability of the shifting phenomenon to both relaxation (creep) and failure response has been taken as strong indication that the time dependence of polymer failure is governed entirely by the viscous deformation processes of the polymer chains. We are thus confronted here with a material which demonstrates that this hypothesis is too simple minded. The other implication may be that the material undergoes internal deformation processes which are fundamentally different from those occurring at small strains and which do not follow the time-temperature superposition principle.

II STRESS ANALYSIS OF CRACK GEOMETRIES

An earlier experimental analysis of the stresses around a crack in a rubber sheet has been amplified to investigate the effect of tearing on the deformation of the crack boundary. It was found that although the initial radius of curvature of the crack tip was quite small (less than 0.002 inches), no tearing occurred until this radius had increased by a factor of 250 while linear elasticity theory would have predicted tearing to occur much sooner. This substantiated through physical measurement a conjecture mentioned in the last semi-annual report[5], namely, that large deformations carry a tremendous reduction in the strain concentration. Figure 2 shows the relation between applied strain and radius of curvature at the crack tip. Loading and unloading occurred after each 10 per cent strain increase: a change in this relation did not occur until about 40 per cent at which strain tip tearing was first observed as crack tip fraying.

Substantial progress has been made on the numerical, finite element analysis of an elliptically perforated rubbersheet undergoing large deformations. In contrast to the earlier calculations involving the integral strain solution based on infinitesimal elasticity[5], this finite element solution incorporated non-linear material behavior through the use of a neo-Hookean stress-strain law. However, just like that integral strain solution, the finite element solution predicts a significant reduction in the stress and strain concentration factor as a function of strain. Figure 3 shows the stress and strain concentration factors at the root of an ellipse having an aspect ratio (ratio of major to minor axis) of 10:1. A more detailed distribution of the maximal principal stress along the direction of the major axis is shown in figure 4 for several values of the overall strain. Note that the distribution of stress changes with strain. This is primarily due to the non-linear material properties.

In order to provide an overall view of the stress field some three-dimensional plots were prepared, three of which are shown in figure 5 for different values of the gross strains.

III CRACK PROPAGATION IN SHEET GEOMETRIES

The basic idea governing the deformation of crack propagation in sheet geometries has been advanced in the last report[5]. These calculations have been carried out to the point where realistic material properties can be substituted for the determination of the velocity-strain relation. The material properties are required in the form of the retardation spectrum. Calculations are now underway to determine this spectrum.*

The theoretical calculations based on linear analyses attempt to calculate from stress analysis and basic material behavior the rate of fracture progression. Agreement or disagreement with experiments will show the degree to which non-linear effects must be considered. Since non-linear affects of the kinematic and material type are concentrated in the crack tip region, it is of interest to investigate whether the crack velocity strain relation can be placed into the frame work of non-linear viscoelastic response behavior. To that end crack propagation tests have been run for some time under various strain histories. The measurements of crack propagation under cyclic strain are being reduced by reading the film records onto IBM cards. The computer smooths the data and determines the velocity and provides a plot of the velocity as well as the Fourier components of the velocity history.

* It will be recalled that the determination of the retardation spectrum is not possible in a straight forward manner using the numerical programs developed earlier[4].

REFERENCES

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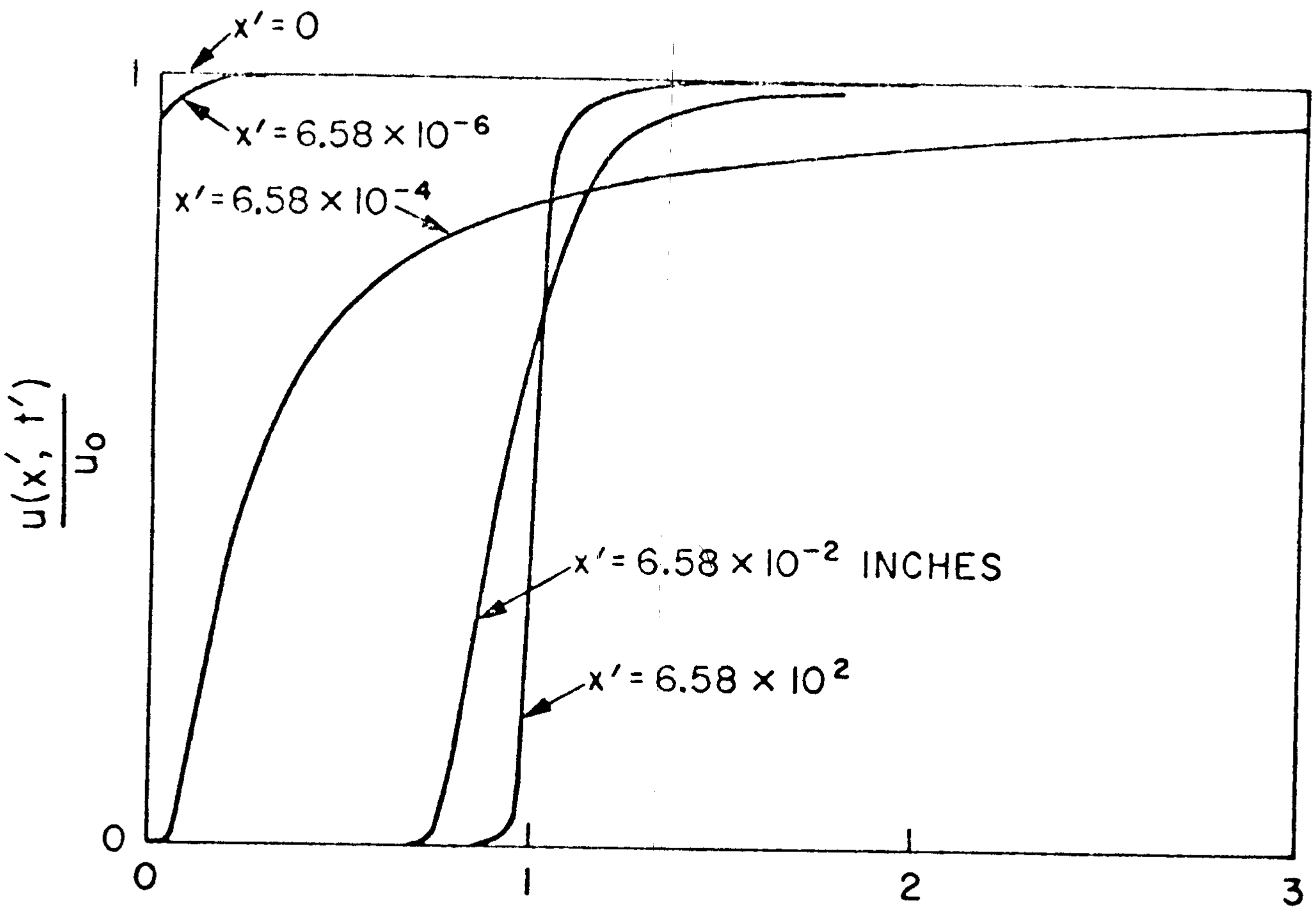

TIME, $\xi = \frac{t - t_g}{t_r - t_g}$

Fig. 1. MOTION HISTORY AT DIFFERENT STATIONS x' AS A FUNCTION OF TIME NORMALIZED BY LIMIT PROPERTIES t_r AND t_g OF THE SOLID

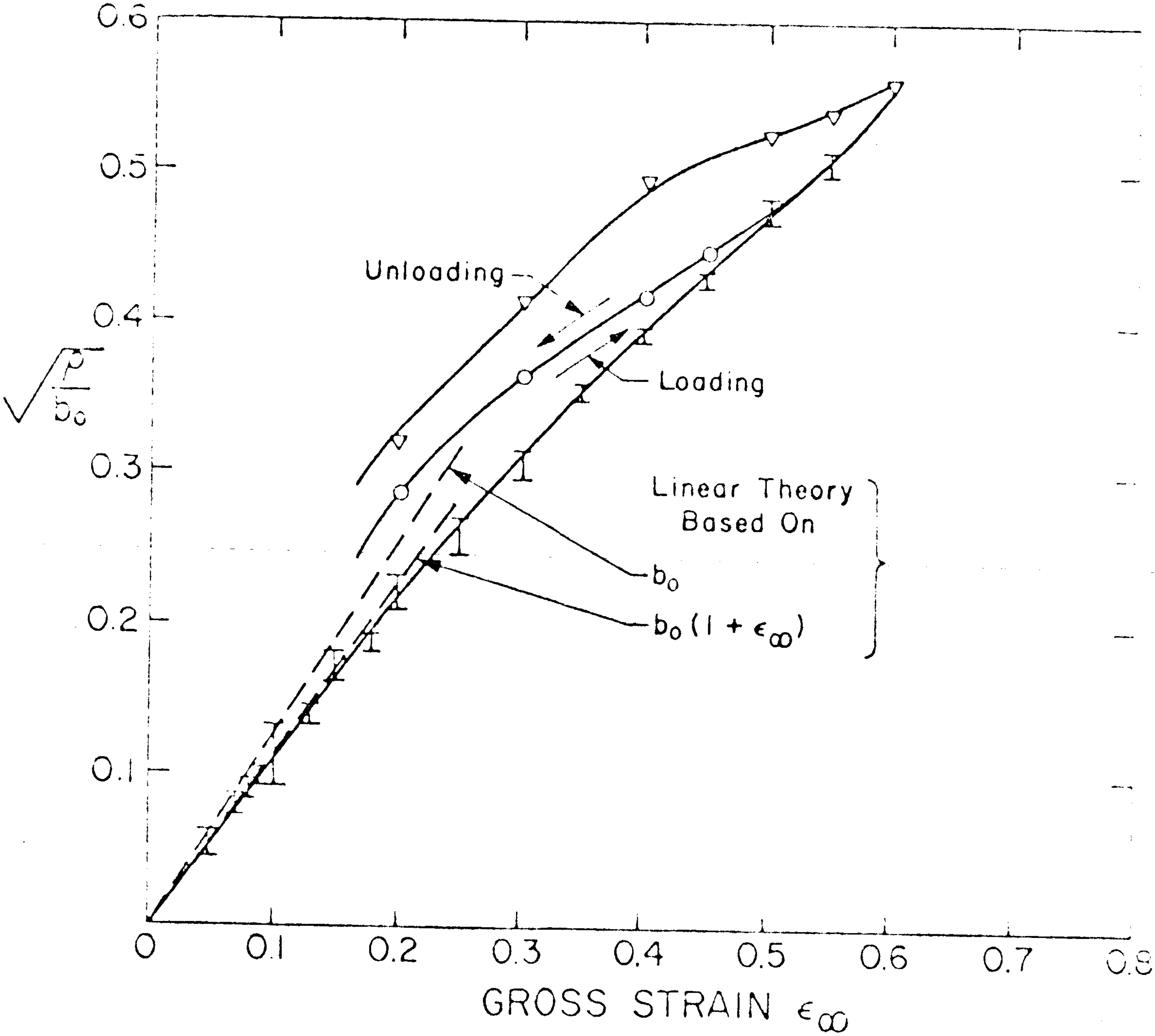

FIG. 2. RADIUS OF CURVATURE vs GROSS STRAIN, EXPERIMENTAL AND CLASSICAL ELASTICITY THEORY

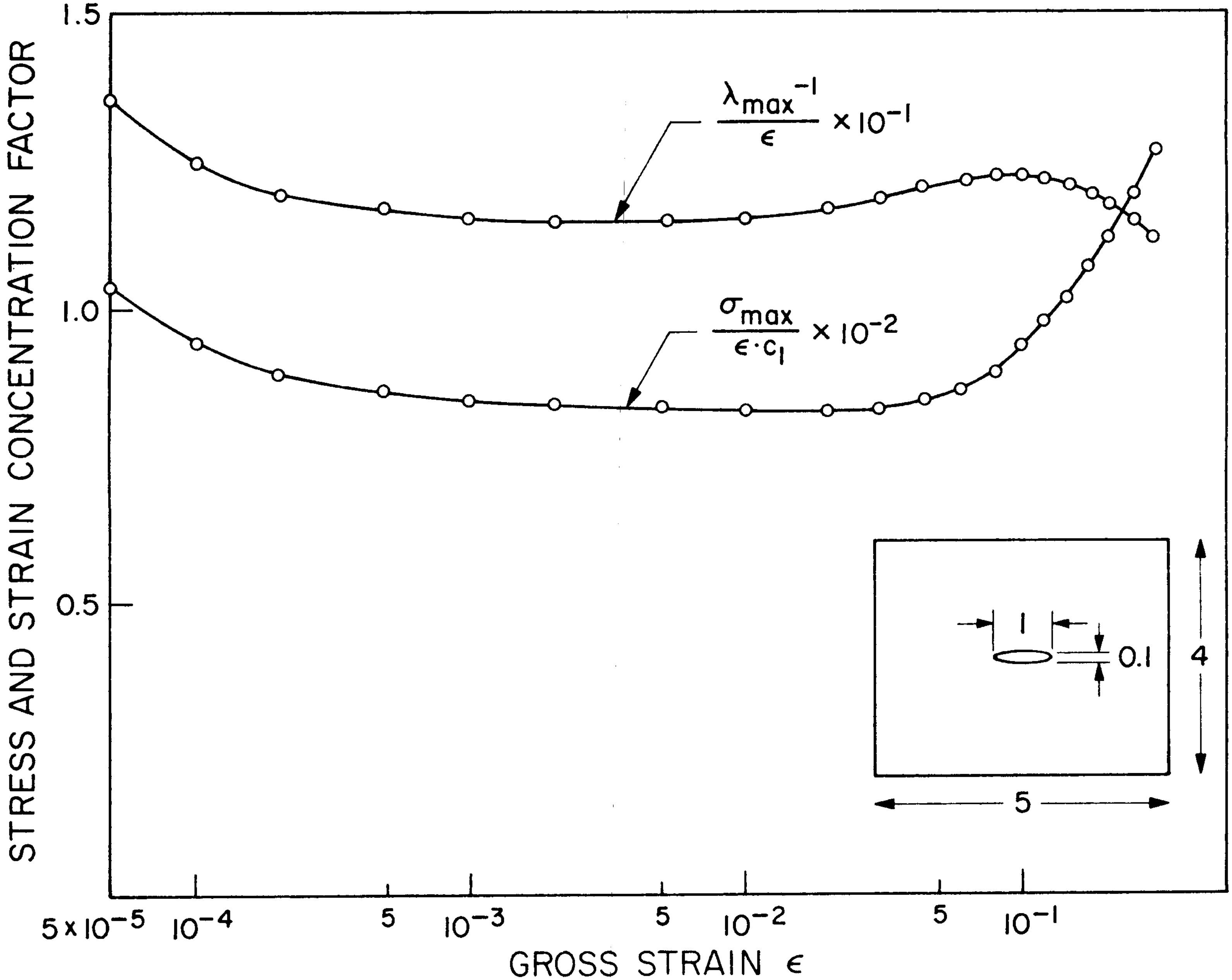

Fig. 3 STRESS AND STRAIN CONCENTRATION IN A 10:1 ELLIPSE AS A FUNCTION OF GROSS STRAIN ϵ.

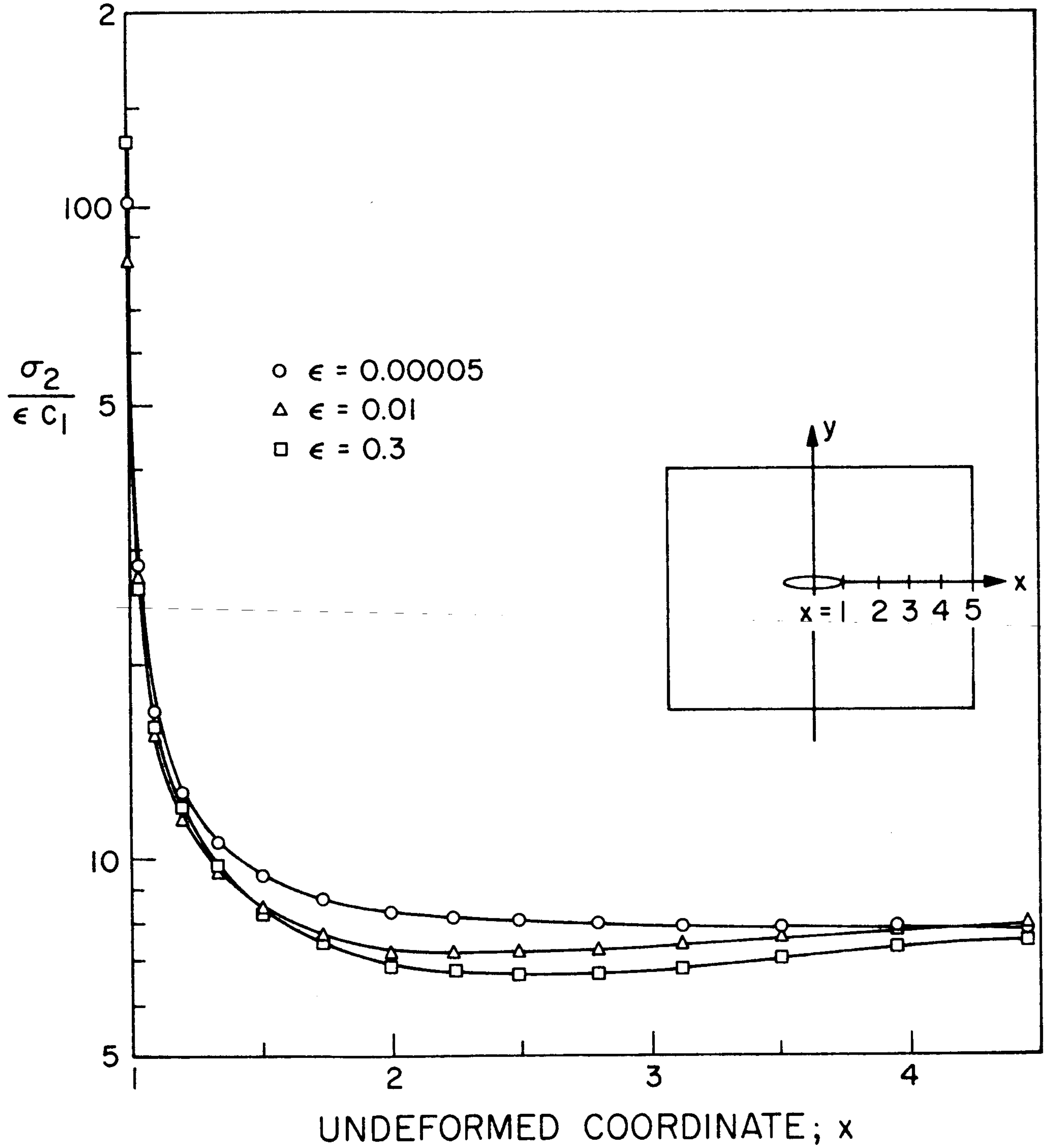

Fig. 4 MAXIMUM STRESS ALONG EXTENSION OF ELLIPSE MAJOR AXIS.

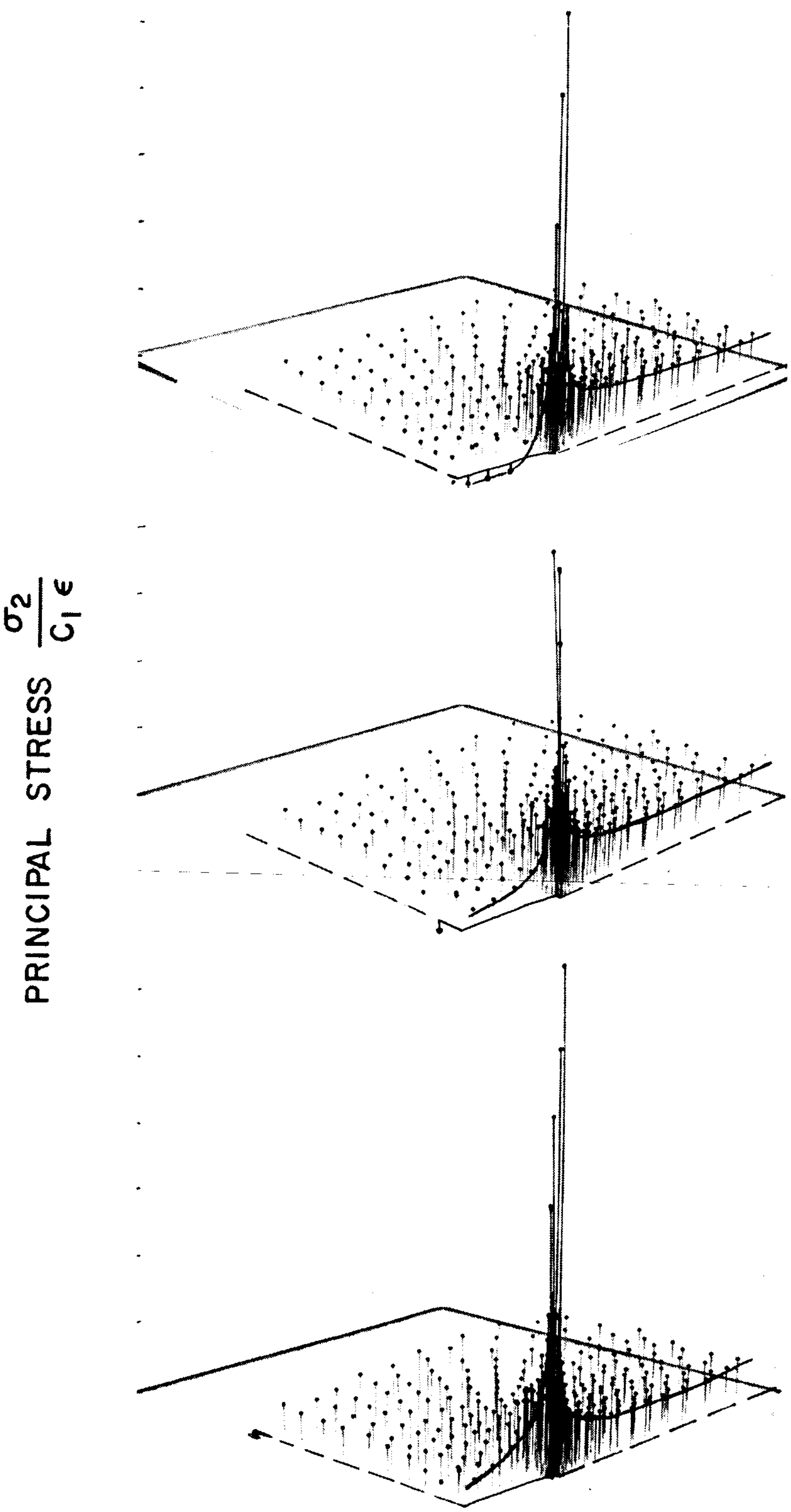

FIG. 5 THREE DIMENSIONAL PLOT OF ONE PRINCIPLE STRESS, σ_2, IN THE VICINITY OF THE ELLIPTIC PERFORATION